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Multispin errors in the optical control of a spin quantum lattice

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We study a spin lattice realized with an array of charged quantum dots and embedded in a cavity. Optically excited polaritons, i.e., exciton-cavity-mixed states, interact with the electron spins in the dots. Linearly polarized excitation induces two-spin and multispin interactions. We discuss how the multispin interaction terms, which represent a source of error for two-qubit quantum gates, can be suppressed using local control of the exciton energy. The exciton spontaneous emission and the photon leakage out of the cavity are taken into account. We show that using detuning conditional phase-shift gates with high fidelity can be obtained. The cavity provides long-range spin coupling and the resulting gate operation time is shorter than the spin decoherence time.

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I. INTRODUCTION

In the last few years there have been great advances toward quantum information processing in the solid state. Yet, there are many theoretical and practical problems that remain to be addressed. In particular, there is not yet a solid-state system for which all the feasibility criteria for quantum computing (i.e., decoherence, reliable one- and two-qubit operations, scalable qubit, initialization, and read out[1]) have been simultaneously demonstrated. Lately, electron spins in semiconductors localized either in low-dimensional nanostructures, i.e., quantum dots (QDs) or in impurities, are increasingly receiving attention as qubits due to their very long decoherence time, which can be of the order of $T_2=3\ \mu s$.[2–4] The long coherence time of the electron spin is due to its weak interaction with the environment, which, however, makes its control more demanding. In this framework, optical techniques are very promising since in this case the control is realized using an optically active ancillary excited state, e.g., a trion state in quantum dots, leading to a control that can be obtained in picoseconds. Optical initialization,[5–7] single qubit measurement,[8,9] and selective one-qubit control of QD spins[10,11] have been already demonstrated. Similar experiments on impurity states have also been carried out.[12,13] The two-qubit control represents a more challenging task. Optically mediated long-range spin-spin interaction in a cavity system has been explored theoretically only for two QDs.[14–18]

In this paper, we show that an array of charged QDs embedded in a planar cavity (see Fig. 1) is a good candidate for a controllable quantum computer. We extend the previous works on polariton-mediated spin coupling[17] to the case of many dots, which leads to the appearance of *multispin* Ising-type coupling terms. Furthermore, we include spontaneous exciton emission and cavity photon leakage. We consider a system in which the energy of the ancillary states on each dot can be controlled independently, for instance, using gates on each dot. We calculate the fidelity of the phase gate of two spins in resonance with the cavity mode corresponding to normal emission and show that by controlling the detuning of the remaining dots, gates with a very small error can be obtained. Errors due to multispin terms in the case of quantum dots directly coupled by wave-function overlap have also been studied recently.[19] The model of multispin coupling is also applicable to similar systems, such as superconducting qubits embedded in a cavity, for which the two-qubit control has been demonstrated in a recent experiment.[20]

II. THEORY

A. Polariton-spin Hamiltonian

Our assumptions for the system studied are the following: (i) the trion energy $\omega_{X,j}$ of each dot can be independently controlled, e.g., by applying a local voltage,[21] (ii) the quantum dots are well separated so there is no direct overlap of the trion wave function, (iii) each dot can be occupied only by one additional exciton, (iv) the heavy-hole and light-hole splitting is large enough that only the heavy-hole exciton is taken into account, and (v) the cavity and the spin lattice are ideal. The role of the cavity is to enhance the range of the interaction between dots[22] and their spins.[17] The long-range coupling is mainly mediated by polariton modes with in-

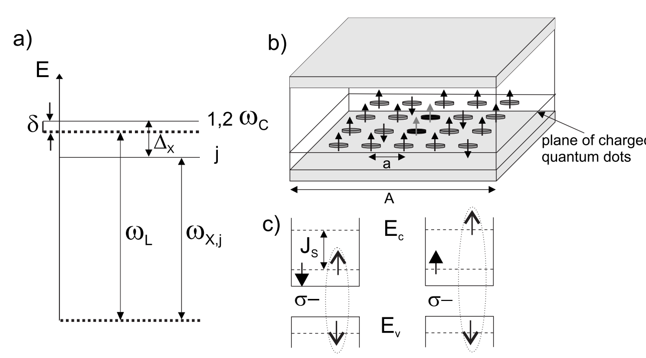

FIG. 1. (a) Energy diagram with cavity energy ω_C, exciton energy $\omega_{X,j}$ and detuning Δ_X (see text for details), and laser frequency ω_L. (b) Scheme of the spin lattice composed by charged quantum dots in a planar cavity. Two dots brought into resonance with the cavity are highlighted. (c) Diagram of allowed spin configurations for a charged dot excited by circularly polarized light. The distance between the trion energy of the two configurations defines an antiferromagnetic spin coupling between the electron spin and the exciton spin (polarization).

plane wave vectors $q \sim 0$; therefore, in the following we neglect cavity modes with $q \neq 0$. The total Hamiltonian describing the spin lattice consists of the three terms: a non-Hermitian polariton term $\hat{H}_P$ describing the interaction of the exciton with the cavity mode, an exciton-spin term $\hat{H}_I$, and a cavity-laser term $\hat{H}_L$ describing the pumping of the cavity mode by an external laser, which is described using the quasimode model.[23] These terms can be written as ($\hbar=1$ throughout the paper)

$$\hat{H}_P = \sum_{\sigma} \left\{ \sum_{j} (\omega_{X,j} - i\Gamma) C^{\dagger}_{j\sigma} C_{j\sigma} + g \sum_{j} (a_{\sigma} C^{\dagger}_{j\sigma} + \text{H.c.}) + (\omega_C - i\kappa) a^{\dagger}_{\sigma} a_{\sigma} \right\},$$

$$\hat{H}_I = \sum_{j} J_S S_{jz} P_{jz},$$

$$\hat{H}_L = \sum_{\sigma} (V_{\sigma} e^{i\omega_L t} a_{\sigma} + \text{H.c.}), \tag{1}$$

where $C^{\dagger}_{j\sigma}$ ($C_{j\sigma}$) is the creation (annihilation) operator of the exciton on the jth dot at position R_j with polarization σ and decay rate Γ, $a^{\dagger}_{\sigma}$ (a_{σ}) is the creation (annihilation) operator of the cavity photon with the energy ω_C (Ref. 24) and cavity leakage rate κ, and g is the dot-photon coupling constant. The use of the non-Hermitian Hamiltonian implies the assumption that the decay processes are only one way,[25] i.e., once the system has decayed it cannot come back. This is a good approximation in our system since the photon emitted in a polariton decay leaves the cavity. The exciton-spin coupling constant J_S in $\hat{H}_I$ is defined as the energy difference between trion states with parallel and antiparallel spins as schematically shown in Fig. 1(c); S_{jz} is the z component of the electron spin in the jth QD; and $P_{jz} = C^{\dagger}_{j\uparrow} C_{j\uparrow} - C^{\dagger}_{j\downarrow} C_{j\downarrow}$ is the operator corresponding to the z component of the exciton polarization. In $\hat{H}_L$, V_{γ} is the laser-cavity coupling constant and ω_L is the frequency of the external laser. A $\sigma+$ ($\sigma-$) polarized photon creates a bright exciton with $\downarrow$ ($\uparrow$) electron spin in the growth (z) direction. For excitons in III-V-confined systems the possible values of the electron spin are $\sigma^{e}_{z} = \pm \frac{1}{2}$ and of the heavy-hole spin are $\sigma^{\text{hh}}_{z} = \mp \frac{3}{2}$. The $\sigma+$ ($\sigma-$) circularly polarized light leads to an effective magnetic field (and higher-order odd terms) in the positive (negative) z direction with the strength proportional to the light intensity.[26] We assume throughout the paper that the light is linearly polarized, which makes all multispin terms of odd order identically zero. This is caused by the fact that these terms would break the time-reversal symmetry,[27] which has to be preserved in the presence of linearly polarized radiation.

B. Multispin Hamiltonian

The effective spin Hamiltonian can be calculated introducing the level shift operator $R(\omega_L)$ as[28]

$$\hat{H}_s = \mathcal{P} R(\omega_L) \mathcal{P} = \mathcal{P} \hat{H}_L \frac{\mathcal{Q}}{\omega_L - \mathcal{Q}(\hat{H}_P + \hat{H}_I)\mathcal{Q}} \hat{H}_L \mathcal{P}, \tag{2}$$

where $\mathcal{P} = \Sigma_{\lambda} |\lambda\rangle\langle\lambda| \otimes |0\rangle\langle 0|$ is the projection operator on the subspace of all spin states λ and no polaritons and $\mathcal{Q} = 1 - \mathcal{P}$. Using the rotating wave approximation and assuming a linearly polarized laser, the cavity-laser term can be rewritten as $\hat{H}_L = V_{\downarrow} a_{\downarrow} + V_{\uparrow} a_{\uparrow} + \text{H.c.}$

We solve first the polariton problem for both polarizations and obtain the polariton states $|\nu\uparrow(\downarrow)\rangle$ satisfying $\hat{H}_P |\nu\uparrow(\downarrow)\rangle = \omega_{\nu} |\nu\uparrow(\downarrow)\rangle$. The polariton states can be written in terms of excitons and the cavity photon as $|\nu\uparrow(\downarrow)\rangle = (\Sigma_j u_{\nu j} C^{\dagger}_{j\uparrow(\downarrow)} + v_{\nu} a^{\dagger}_{\uparrow(\downarrow)})|0\rangle$. The coefficients $u_{\nu j}$ represent the projection of the polariton state ν on the exciton state localized at the jth dot. Similarly, v_{ν} represents the projection of the polariton state on the cavity photon. The coefficients $u_{\nu j}$ and v_{ν} are called Hopfield coefficients[29] in the polariton literature. Using these coefficients the Hamiltonian $\hat{H}_s$ projected on the spin basis $|\lambda\rangle$ reads

$$H^{\lambda\lambda'}_{s} = \sum_{\mu\nu} v_{\mu} v^{*}_{\nu} \sum_{\sigma=\uparrow,\downarrow} \frac{V^2_{\sigma}}{2} \langle \mu\sigma | \langle \lambda | \{\omega_L - (\hat{H}_P + \hat{H}_I)\}^{-1} | \lambda' \rangle | \nu\sigma \rangle. \tag{3}$$

The off-diagonal terms $\langle \lambda | \{\omega_L - (\hat{H}_P + \hat{H}_I)\}^{-1} | \lambda' \rangle$ are zero since all spin-dependent terms are proportional to S_z. This allows us to calculate the eigenenergies of the Hamiltonian $\hat{H}_s$ (3) exactly by inversion of the matrix. Perturbation theory can also be applied by expanding the resolvent as

$$\frac{1}{\omega_L - (\hat{H}_P + \hat{H}_I)} = \frac{1}{\omega_L - \hat{H}_P} + \frac{1}{\omega_L - \hat{H}_P} \hat{H}_I \frac{1}{\omega_L - \hat{H}_P} + \frac{1}{\omega_L - \hat{H}_P} \hat{H}_I \frac{1}{\omega_L - \hat{H}_P} \hat{H}_I \frac{1}{\omega_L - \hat{H}_P} + \cdots. \tag{4}$$

Light with circular $\sigma+$ ($\sigma-$) polarization contributes with terms $\hat{H}_I \sim J_S$ ($\sim -J_S$) according to Eq. (1); consequently, terms with odd terms $\sim J^{2n+1}_{S}$ compensate exactly for linearly polarized light. After some straightforward algebra we can rewrite the effective spin Hamiltonian as

$$\hat{H}_s = \tilde{J}^{(0)} + \sum_{i>j} \tilde{J}^{(2)}_{ij} S_{iz} S_{jz} + \sum_{i>j>k>l} \tilde{J}^{(4)}_{ijkl} S_{iz} S_{jz} S_{kz} S_{lz} + \ldots, \tag{5}$$

where the coupling constants are renormalized to take into account multiple scattering, e.g.,

$$\tilde{J}^{(2)}_{12} = J^{(2)}_{12} + J^{(2)}_{21} + \sum_{i\mathbf{P}} J^{(4)}_{\mathbf{P}(12ii)} + \sum_{ij\mathbf{P}} J^{(6)}_{\mathbf{P}(12iijj)} + \cdots, \tag{6}$$

where $\mathbf{P}$ indicates a permutation of all the indices. The multispin coupling constants $J^{(n)}_{i_1 \ldots i_n}$ can be expressed as

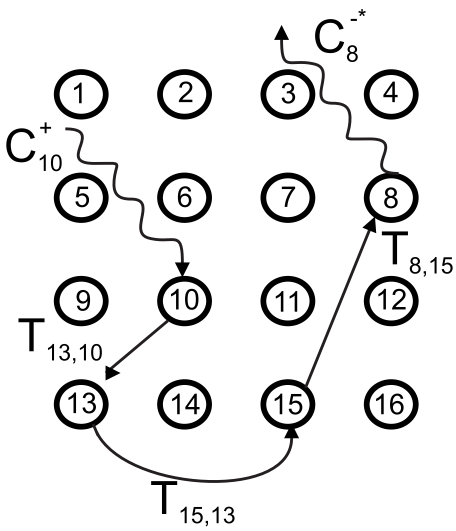

FIG. 2. Diagram illustrating multiple-scattering events that lead to a multispin coupling $J^{(4)}_{8,15,13,10}$ as derived in Eq. (7).

$$J^{(n)}_{i_1\cdots i_n} = J_S^n V_{\mathrm{LP}}^2 (\mathbf{C}^-_{i_1})^* \mathbf{T}_{i_1 i_2} \cdots \mathbf{T}_{i_{n-1} i_n} \mathbf{C}^+_{i_n} \tag{7}$$

in terms of the photon-exciton coupling function $\mathbf{C}_i^{+(-)}$ and exciton interdot transfer amplitudes $\mathbf{T}_{ij}$ defined as (see Fig. 2)

$$\mathbf{C}_i^{+(-)} = \sum_\mu \frac{v_\mu u^*_{\mu i}}{\omega_L - \omega_\mu}, \quad \mathbf{T}_{ij} = \sum_\mu \frac{u_{\mu i} u^*_{\mu j}}{\omega_L - \omega_\mu}, \tag{8}$$

where $V_{\mathrm{LP}}^2 = \frac{V_\uparrow^2 + V_\downarrow^2}{2}$ is the effective light-polariton coupling constant. Note that due to the non-Hermitian nature of the polariton Hamiltonian the polariton energies ω_μ are complex quantities with the imaginary part $\gamma_\mu = \mathrm{Im}\, \omega_\mu$ representing the polariton linewidth.

Let us now consider two dots labeled by {1,2} in resonance with the lowest cavity mode, i.e., $\omega_{X,1(2)} = \omega_C$. The laser is detuned below the cavity mode as $\omega_L = \omega_C - \delta$, where δ is the *laser detuning*. The remaining quantum dots are detuned: $\omega_{X,1(2)} - \omega_{X,j\neq 1,2} = \Delta_X$, where Δ_X is the *exciton detuning* [schematically shown in Fig. 1(a)]. Dots shifted off resonance by a dc Stark shift will also have a weaker light-dot coupling g due to the decrease in the electron-hole overlap. However, in order to have a conservative estimate of the error we neglect this effect.

C. Fidelity of a conditional phase-shift gate

We will now estimate the error in the implementation of a conditional phase-shift gate due to multispin interaction terms. The conditional phase gate (PG) is a universal two-qubit gate, i.e., it can realize universal quantum computation when combined with single qubit operations.[30] In the computational basis $\{|\downarrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\uparrow\uparrow\rangle\}$ the PG can be written as a diagonal matrix with elements $U_{\mathrm{PG}} = \{1,1,1,-1\}$. With an Ising-type interaction between two spins $\sim S_{z1}S_{z2}$, the following sequence[31] gives the PG: $U_{\mathrm{PG}} = e^{-i\pi/4}[S_{z1} + S_{z2}][-2S_{z1}S_{z2}]$ with $[P] = e^{i\pi/2P}$. A quantitative measure of the gate quality can be given using the gate *fidelity*[32] defined

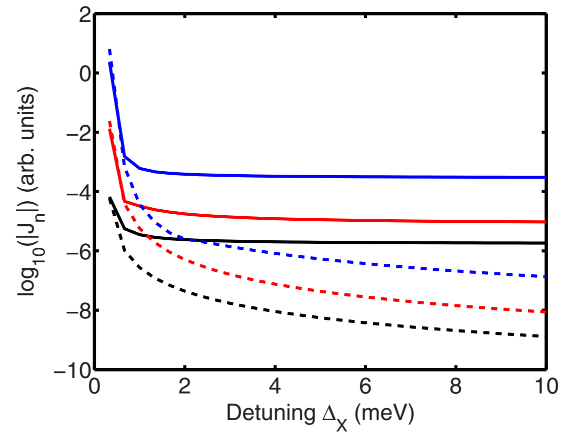

FIG. 3. (Color online) Logarithmic plot of $J_R^{(n)}$ (solid) and $J_O^{(n)}$ (dashed) as a function of the exciton detuning Δ_X in a 3×3 array of charged QDs with $g = 100$ μeV, $J_S = 0.43$ meV, $V_{\mathrm{LP}} = 10$ μeV, and $\delta = 50$ μeV. From bottom to top: $n=2$ (black), $n=4$ (red), and $n=6$ (blue).

as $\mathcal{F} = \overline{|\langle\Psi|U_I^\dagger U_R|\Psi\rangle|^2}$, where U_I is the ideal gate matrix and U_R is the real gate matrix, i.e., the one that includes the effects of multispin terms. Ψ is an arbitrary initial pure state and $\overline{|\langle\Psi|\cdot|\Psi\rangle|^2}$ indicates averaging over all pure initial states. Working in the basis of the full spin-Hamiltonian eigenstates $\{\phi_i\}$ (with 2^{N_D} states), we can define an eigenvector fidelity as $\mathcal{F}_i = \langle\phi_i|U_I^\dagger U_R|\phi_i\rangle$. In order to investigate *only* the role of multispin terms, the single qubit operations $[S_{z1}+S_{z2}]$ are assumed to be ideal. Consequently, the fidelity is influenced only by the Ising part of the whole gate $[-2S_{z1}S_{z2}]$ and since the total Hamiltonian does not allow for spin-flip processes, the fidelity can then be expressed as $\mathcal{F} = |\frac{1}{N_D}\Sigma_i \mathcal{F}_i|^2$. In order to calculate the fidelity, we calculate the time evolution operator $U(t_C) = \exp\{-iH_s t_C\}$ with a time $t_C = \frac{\pi}{2|\tilde{J}^{(2)}_{12}|}$ to obtain maximal fidelity. The Gedanken gate sequence can be described as follows: (i) single qubit operations are performed on two selected dots {1,2} (e.g., by optical control[10,11]), then (ii) they are brought adiabatically into the resonance with the cavity by controlling the exciton energy with local electric field, (iii) the laser is switched on adiabatically for a time t_C, and (iv) dots are brought back into the off-resonant state.

However, the computation can be spoiled either by the exciton decay or by the photon leakage. In order to estimate the effect of the exciton spontaneous emission and the photon leakage in the spin-spin coupling within the polariton picture ($\kappa, \Gamma \ll g$) we consider the first two terms in Eq. (5) (i.e., we neglect contributions beyond the second order in J_S for a moment) and compare the real part of the second term, which gives the effective spin-spin interaction $J^{(2)}_{12}$ to the lowest order, and the imaginary part of the first term, which gives the decay rate independent of the spin state. The latter describes the process in which the polariton leaves the cavity without any interaction with the spins, which would fail the gate operation (even if it does not spoil the spin coherence). Introducing for simplicity a constant polariton linewidth γ we obtain

$$\mathrm{Re}\, J^{(2)}_{12} \sim \frac{V_{\mathrm{LP}}^2 J_S^2 |v|^2 |u|^2}{(\delta^2+\gamma^2)^3} (\delta^3 - 3\delta\gamma^2),$$

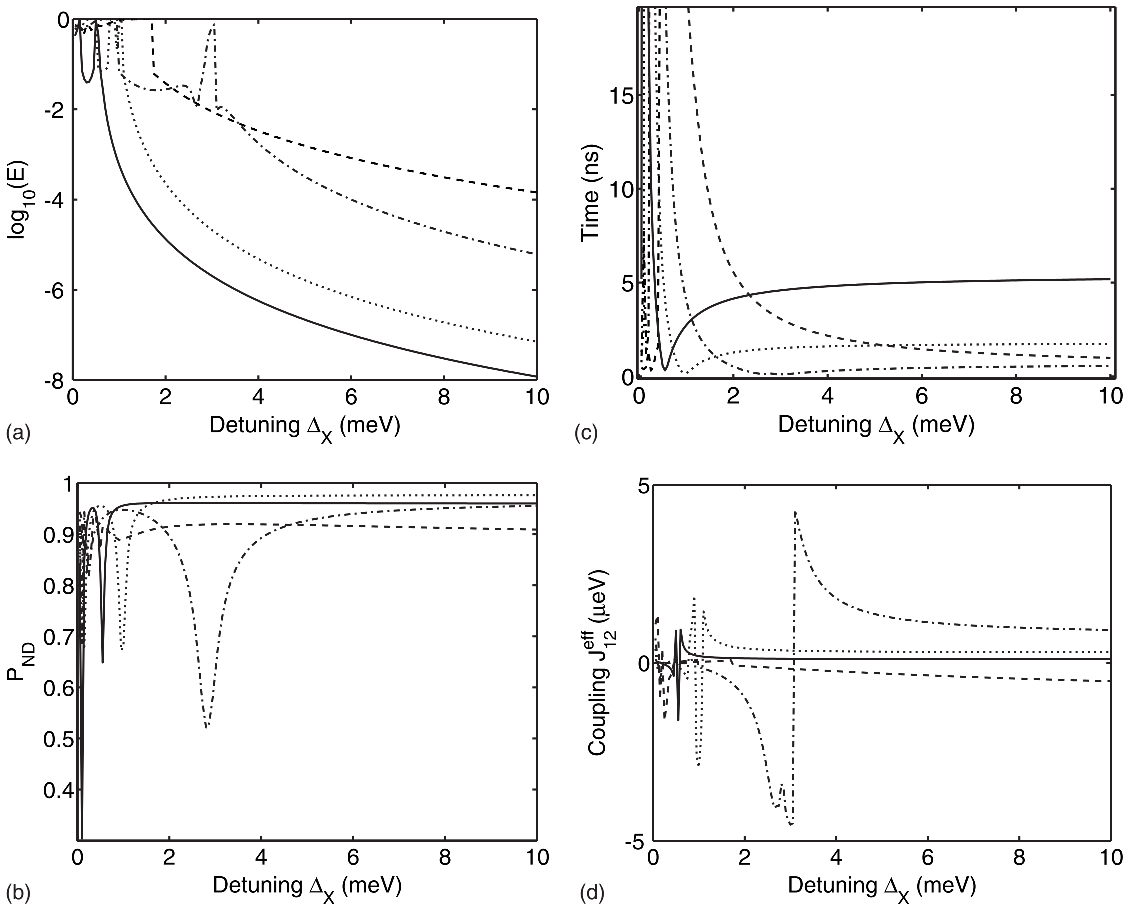

FIG. 4. (a) Logarithmic plot of the error $\mathcal{E}$, (b) the probability of no state decay P_{ND} during the computation, (c) the time of the computation t_C, and (d) the effective coupling constant $\tilde{J}_{12}^{(2)}$ as a function of the exciton detuning Δ_X. The phase gate between two dots in an array of nine charged QDs, $g=100$ μeV, $\delta=50$ μeV, $V_{\mathrm{LP}}=10$ μeV, $J_s=0.86$ meV (dashed), $J_S=0.43$ meV (dashed-dotted), $J_S=0.22$ meV (dotted), and $J_S=0.11$ meV (solid).

$$\mathrm{Im}\, J^{(0)} \sim -\frac{V_{\mathrm{LP}}^2|v|^2}{(\delta^2+\gamma^2)}\gamma, \tag{9}$$

where $|v|^2$ and $|u|^2$ are the photon and the exciton parts of the polariton (Hopfield coefficients), which are of order of one. Note that in the limit $g\to 0$ we have for the Hopfield coefficients in this expression $u\to 0$ and $v\to 1$, i.e., the coupling of the cavity photon and the exciton is necessary for the spin coupling. Together with the condition to have a small probability of the polariton emission during the gate operation, $\frac{\mathrm{Im}\, J^{(n)}}{\mathrm{Re}\, J^{(n)}}\sim\frac{\gamma}{\delta}\ll 1$ in the limit of $\delta\gg\gamma$, this translates into

$$\frac{\mathrm{Im}\, J^{(0)}}{\mathrm{Re}\, J_{12}^{(2)}} \sim \frac{\gamma\delta}{J_S^2} \ll 1, \tag{10}$$

which requires $J_S\gg\delta$. These are rather strict demands taking into account that the limit of the weak cavity-laser coupling is assumed, $V_{\mathrm{LP}}\ll\delta$, and that for a fast computation the coupling V_{LP} can be decreased only within a limited range since $t_C\sim J^{-1}\sim V_{\mathrm{LP}}^{-2}$. Within the quantum jump approach, the nonzero probability of state decay can be calculated as $P_D=1-P_{\mathrm{ND}}=1-\frac{1}{N_D}\mathrm{tr}\{|U(t_C)|\}$ [$U(t_C)$ is a diagonal matrix] in our case. Assuming that there has not been any decay, the $U(t_C)$ is then renormalized to one for the calculation of the fidelity.[25] However, from the experimental point of view this approach is justified only if the polariton decay can be directly measured, which is a very demanding task.

In order to preserve adiabaticity, a similar condition as in Ref. 33 can be derived for a Gaussian pulse of length τ. This condition reads $\tau\gg\frac{V_{\mathrm{LP}}}{\delta'^2}$, where δ' is the *renormalized* laser detuning, which is of the order of J_S. The renormalization of the detuning is due to the shift of the polariton energy induced by the spin coupling. Since in our case the relation $V_{\mathrm{LP}}\sim\frac{J_S}{100}$ holds (see Figs. 4 and 5), it is sufficient that $\tau\sim\frac{1}{10\delta'}$. This poses generally no limits on the time of the computation because $\frac{1}{\delta'}<1$ ns, which is shorter than our calculated time of the computation, as discussed below. In the related problem of indirect optically induced interaction between localized spins the pulse shape and the decay rate were explicitly included and results confirmed the mentioned criteria.[34]

III. RESULTS AND DISCUSSION

Most of the parameters used in the calculations are given in the respective captions of the figures. The maximum ex-

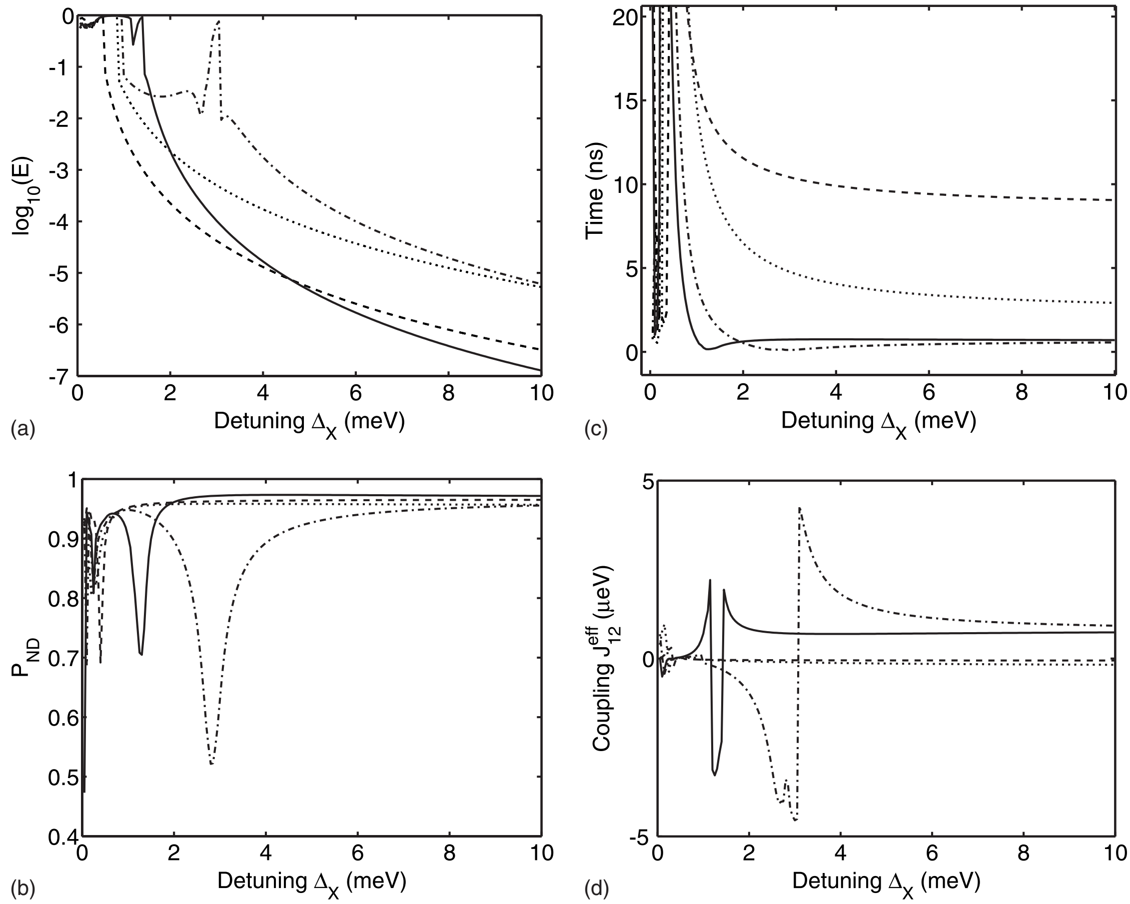

FIG. 5. (a) Logarithmic plot of the error $\mathcal{E}$, (b) the probability of no state decay P_{ND} during the computation, (c) the time of the computation t_C, and (d) the effective coupling constant $\tilde{J}_{12}^{(2)}$ as a function of the exciton detuning Δ_X. The phase gate between two dots in an array of nine charged QDs, $g=100$ μeV, $J_S=0.86$ meV, $V_{\mathrm{LP}}=10$ μeV, $\delta=200$ μeV (dashed), $\delta=100$ μeV (dotted), $\delta=50$ μeV (dashed-dotted), and $\delta=25$ μeV (solid).

citon detuning is set to $\Delta_X=10$ meV, which is about the upper limit for a Stark shift that can be obtained in current experiments. For simplicity, we assume the same exciton and photon decay rates of $\Gamma=\kappa=2$ μeV, which requires a very high Q ($\sim 10^6$) cavity, but is of the right order of magnitude for self-assembled QDs. In the numerical calculation we consider a finite system with nine dots.

First, the dependence of *real* parts of different multispin terms on the detuning is shown in Fig. 3 where we separate the terms that involve the two dots nearly resonant with the cavity from the other terms related to the off-resonant dots. We plot $J_{12}+J_{21}$ (solid black) and $\Sigma_{ij\notin\{1,2\}}|J_{ij}|$ (black dashed) for $n=2$ spin terms. For multispin terms ($n=4$, $n=6$) the contributions that renormalize the effective coupling between the two resonant dots 1 and 2 ($J_R^{(n)}$) are separated from contributions that involve only the off-resonant dots strongly detuned from the cavity ($J_O^{(n)}$). For instance, for $n=4$ the resonant (off-resonant) terms are defined as $J_R^{(4)}=\Sigma_{\mathbf{P}}|J^{(4)}_{\mathbf{P}(1122)}|+\Sigma_{i\mathbf{P}}|J^{(4)}_{\mathbf{P}(12ii)}|$ ($J_O^{(4)}=\Sigma_{ijkl}|J^{(4)}_{ijkl}|-|J_R^{(4)}|$). This definition enables us to better estimate the contribution of the off-resonant terms. In fact, even if the magnitude of the individual terms $J^{(n)}_{i_1 \cdot\cdot i_n}$ is small there is an enhancement due to the large number of n-dot combinations [$\sim\binom{n}{N_D}$]. Note that although the magnitude of the resonant term increases with n since $J_R^{(n)}\sim\frac{J_S^n V_{\mathrm{LP}}^2}{\delta^{(n+1)}}$ and the ratio $J_S \gg \delta$ dominates over $\delta \gg V_{\mathrm{LP}}$, there is only a weak dependence on the exciton detuning for the resonant terms and a strong decrease for the off-resonant terms ($J^{(n)}\sim\frac{J_S^n V_{\mathrm{LP}}^2}{\Delta_X^{(n-1)}}$) as expected from the form of the coupling in Eqs. (7) and (8).

Second, we show in Figs. 4 and 5 the calculated error $\mathcal{E}=1-\mathcal{F}$, the probability P_{ND} of no state decay during a gate with time t_C (no quantum jump), and the effective spin-spin coupling $\tilde{J}_{12}^{(2)}$ as a function of the exciton detuning for different spin energy J_S and laser detuning δ. In both cases, we may notice that the error tend to decrease with increasing exciton detuning Δ_X since at larger detunings only the two selected dots {1,2} remain in resonance with the cavity and the multispin coupling with the other dots is suppressed. The minimal value of the error $\mathcal{E}$ for maximal exciton detuning Δ_X depends (i) on the laser detuning δ, which decreases the effective polariton population during the computation and thus the effect of decays [Fig. 5(a)], and (ii) on the spin energy J_S [Fig. 4(a)]. The spins of the off-resonant dots form spin multiplets separated by $2J_S$ and their maximal energy is approximately $\omega_{X,j}+\frac{7}{2}J_S$, which renormalizes the effective exciton detuning as $\tilde{\Delta}_X=\Delta_X-\frac{7}{2}J_S$.

Another clear trend at large Δ_X is the decrease in the computation time with increasing the spin energy [Fig. 4(c)] and decreasing the laser detuning [Fig. 5(c)], which is caused by the characteristic dependence of the coupling $\tilde{J}_{12}^{(2)} \sim f(\frac{J_S}{\delta})$, where f is a monotonic increasing function. We note that for sufficiently strong exciton detuning, the operation times are $t_C < 5$ ns. Thus, they are shorter than the spin decoherence time T_2, which is of orders of at least μs. Moreover, it turns out that the optimal way to decrease the computation time t_C for large Δ_X within the assumptions $\gamma \ll \delta \ll J_S$ and $V_{\mathrm{LP}} \ll \delta'$ (detuning with respect to the polariton-spin levels) is to increase the spin energy J_S. However, if the ratio $\frac{\delta}{J_S}$ becomes too small then the spin coupling will decrease again [$\tilde{J} \sim f'(\frac{\delta}{J_S})$, where f' is an increasing monotonic function] as shown in Fig. 4 (dashed). The computation time t_C can be optimized by changing the quantum dot shape and size. We note that the small magnitude of the coupling strength $\sim \mu$eV (consequently the relatively long computation time $t_C \sim V_{\mathrm{LP}}^{-2}$) compared to a similar system studied in Ref. 17 is predominantly due to our smaller laser-cavity coupling $V_{\mathrm{LP}} = 10$ μeV, which is a more conservative choice compared with the value of $V_{\mathrm{LP}} = 0.7$ meV in Ref. 17, and satisfies the condition $V_{\mathrm{LP}} \ll \delta'$ much better.

The behavior of the probability P_{ND} is more complicated since it is affected by the strength of the spin interaction $J_{12}^{(2)}$, by the time of computation t_C, and by the distance between the laser energy ω_L (or detuning δ) and the energy of the trions $\omega_{X,1} \pm \frac{J_S}{2}$. In general, the probability of no decay P_{ND} should increase by decreasing the laser detuning δ as follows from Eq. (10). On the other hand, since the condition $\delta < J_S$ has to be fulfilled, the probability of the no decay increases by increasing J_S (up to the certain value of $\frac{\delta}{J_S}$).

Now we turn our attention to the resonant features in the effective coupling $\tilde{J}_{12}^{(2)}$ as shown in Figs. 4(d) and 5(d). A similar resonantlike behavior has been already predicted in the case of the spins localized by impurity centers in a semiconductor host.[26] Here, the features are caused by spin multiplets, which for small values of the exciton detuning Δ_X are mixed with the spins of the resonant dots through $\tilde{J}_{12ij}^{(4)}$ and higher-order terms. Since each spin multiplet has its own decay rate γ and characteristic dependence on the spin energy J_S, it can happen that the laser energy is close to one of the spin multiplet energies. When this happens, it (i) increases the effective spin-spin coupling, (ii) increases the decay rate, (iii) decreases the fidelity, and (iv) decreases the time of computation as can be nicely seen in Figs. 4 and 5, especially for the dashed-dotted line. These resonances prevent a single monotonic decrease in the error with increasing exciton detuning Δ_X. Moreover, we note that in the resonant case (or for the smallest detuning $\delta = 25$ μeV) the condition of validity of our approach $V_{\mathrm{LP}} \ll \delta'$ is not strictly satisfied and higher-order terms should be taken into account. These higher-order terms represent the probability of having at least two photons in the cavity, which will further increase the error probability.

Note that the technologically accessible maximal value of the individual dot detuning is limited by the interdot separation, since for dots that are too close, it is difficult to place a selective strong gate voltage. The dependence on the lattice constant can be included easily into our model by taking into account many cavity modes with $q \neq 0$. Indeed, we have done several of such calculations (which are much more numerically demanding) and we have studied the fidelity $\mathcal{F}$ and probability of no state decay P_{ND} as a function of the lattice constant. In principle, this information can be used to select an optimal lattice constant.

Finally, we note that for large enough detunings, the present approach can be applied to inhomogeneous and randomly distributed QDs, which has been recently studied.[35]

IV. CONCLUSIONS

We have studied an array of charged quantum dots embedded in a planar cavity. We have shown that optical excitation can be used to control the spins and implement quantum gates. The optical excitation couples many dots in the quantum computer, and multispin interaction terms beyond the ideal two-spin interaction are generated. We have shown that the multispin terms can induce errors in the gate operation even if their value is small, due to their multiplicity. These errors can be corrected by a local control of the excitonic resonance on each dot. In the control scheme we also include exciton spontaneous emission and photon leakage out of the planar cavity. The present control scheme can be applied to other similar solid-state systems, e.g., superconducting qubits embedded in a cavity.

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